| Question | Scheme | Marks | AOs |
|---|---|---|---|
| **1(a)** | $u_3 = £20\,000 \times 1.08^2 = (£)\,23328*$ | B1* | 1.1b |
| | | **(1)** | |
| **(b)** | $20\,000 \times 1.08^{n-1} > 65\,000$ | M1 | 1.1b |
| | $1.08^{n-1} > \frac{13}{4} \Rightarrow n-1 > \frac{\ln(3.25)}{\ln(1.08)}$ or e.g. $1.08^{n-1} > \frac{13}{4} \Rightarrow n-1 > \log_{1.08}\left(\frac{13}{4}\right)$ | M1 | 3.1b |
| | Year 17 | A1 | 3.2a |
| | | **(3)** | |
| **(c)** | $S_{20} = \frac{20\,000(1-1.08^{20})}{1-1.08}$ | M1 | 3.4 |
| | Awrt (£) 915 000 | A1 | 1.1b |
| | | **(2)** | |
| | | | **(6 marks)** |
| | **Notes** | | |

(a)
B1*: Uses a correct method to show that the Profit in Year 3 will be £23 328. Condone missing units
E.g. $£20\,000 \times 1.08^2$ or $£20\,000 \times 108\% \times 108\%$

This may be obtained in two steps. E.g $\frac{8}{100} \times 20000 = 1600$ followed by $\frac{8}{100} \times 21600 = 1728$ with the calculations $21600 + 1728 = 23328$ seen.
Condone calculations seen as $8\%$ of 20000 = 1600.
This is a show that question and the method must be seen.
It is not enough to state Year 1 = £21 600, Year 2 = £ 23 328

(b)
M1: Sets up an inequality or an equation that will allow the problem to be solved.
Allow for example $N$ or $n$ for $n-1$. So award for $20\,000\times1.08^{n-1} > 65\,000$,
$20\,000\times1.08^{n} = 65\,000$ or $20\,000\times(108\%)^{n} \geqslant 65\,000$ amongst others.
Condone **slips** on the 20 000 and 65 000 but the 1.08 o.e. must be correct
M1: Uses a correct strategy involving logs in an attempt to solve a type of equation or inequality of the form seen above. It cannot be awarded from a sum formula
The equation/inequality must contain an index of $n-1$, $N$, $n$ etc.
Again condone **slips** on the 20 000 and 65 000 but additionally condone an error on the 1.08, which may appear as 1.8 for example
E.g. $20\,000\times1.08^{n} = 65\,000 \Rightarrow n\log1.08 = \log\dfrac{65000}{20000} \Rightarrow n = \ldots$
E.g. $2000\times1.8^{n} = 65\,000 \Rightarrow \log 2000 + n\log1.8 = \log 65000 \Rightarrow n = \ldots$
A1: Interprets their decimal value and gives the correct year number. Year 17

The demand of the question dictates that solutions relying entirely on calculator technology are not acceptable, BUT allow a solution that appreciates **a correct term** formula or the entire set of calculations where you may see the numbers as part of a larger list
E.g. Uses, or implies the use of, an acceptable calculation and finds value(s)
for M1: $(n=16) \Rightarrow P = 20\,000\times1.08^{15}$ =awrt 63400 **or** $(n=17) \Rightarrow P = 20\,000\times1.08^{16}$ = awrt 68500

M1: $(n=16) \Rightarrow P = 20\,000\times1.08^{15}$ =awrt 63400 **and** $(n=17) \Rightarrow P = 20\,000\times1.08^{16}$ = awrt 68500
A1: 17 years following correct method and both M's

(c)
M1: Attempts to use the model with a **correct** sum formula to find the total profit for the 20 years.
You may see an attempt to find the sum of 20 terms via a list. This is acceptable provided there are 20 terms with $u_n = 1.08\times u_{n-1}$ seen at least 4 times and the sum attempted.
Condone a slip on the 20 000 (e.g appearing as 2 000) and/or a slip on the 1.08 with it being the same "$r$" as in (b) . Do not condone 20 appearing as 19 for instance
A1: awrt £915 000 but condone missing unit

The demand of the question dictates that all stages of working should be seen. An answer without working scores M0 A0

| Question | Scheme | Marks | AOs |
|---|---|---|---|
| 2 | $a=\left(\frac{3}{4}\right)^2$ or $a=\frac{9}{16}$ or $r=-\frac{3}{4}$ | B1 | 2.2a |
| | $\sum_{n=2}^{\infty}\left(\frac{3}{4}\right)^n\cos(180n)^\circ=\frac{\frac{9}{16}}{1-\left(-\frac{3}{4}\right)}=...$ | M1 | 3.1a |
| | $=\frac{9}{28}$ * | A1* | 1.1b |
| | | **(3)** | |
| | **Alternative 1:** | | |
| | $\sum_{n=1}^{\infty}\left(\frac{3}{4}\right)^n\cos(180n)^\circ=\frac{-\frac{3}{4}}{1-\left(-\frac{3}{4}\right)}=...$ or $r=-\frac{3}{4}$ | B1 | 2.2a |
| | $\sum_{n=2}^{\infty}\left(\frac{3}{4}\right)^n\cos(180n)^\circ=-\frac{3}{7}-\left(-\frac{3}{4}\right)$ | M1 | 3.1a |
| | $=\frac{9}{28}$ * | A1* | 1.1b |
| | **Alternative 2:** | | |
| | $\sum_{n=2}^{\infty}\left(\frac{3}{4}\right)^n\cos(180n)^\circ=\left(\frac{3}{4}\right)^2-\left(\frac{3}{4}\right)^3+\left(\frac{3}{4}\right)^4-...$ | B1 | 2.2a |
| | $=\left(\frac{3}{4}\right)^2+\left(\frac{3}{4}\right)^4+...-\left(\frac{3}{4}\right)^3-\left(\frac{3}{4}\right)^5-...$ <br> $\left(\frac{3}{4}\right)^2+\left(\frac{3}{4}\right)^4+...=\left(\frac{3}{4}\right)^2\left(\frac{1}{1-\left(\frac{3}{4}\right)^2}\right)$ or $-\left(\frac{3}{4}\right)^3-\left(\frac{3}{4}\right)^5-...=-\left(\frac{3}{4}\right)^3\left(\frac{1}{1-\left(\frac{3}{4}\right)^2}\right)$ <br> $\sum_{n=2}^{\infty}\left(\frac{3}{4}\right)^n\cos(180n)^\circ=\left(\frac{3}{4}\right)^2\left(\frac{1}{1-\left(\frac{3}{4}\right)^2}\right)-\left(\frac{3}{4}\right)^3\left(\frac{1}{1-\left(\frac{3}{4}\right)^2}\right)$ | M1 | 3.1a |
| | $=\frac{9}{28}$ * | A1* | 1.1b |
| | **Alternative 3:** | | |
| | $\sum_{n=2}^{\infty}\left(\frac{3}{4}\right)^n\cos(180n)^\circ=S=\left(\frac{3}{4}\right)^2-\left(\frac{3}{4}\right)^3+\left(\frac{3}{4}\right)^4-...$ | B1 | 2.2a |
| | $=\left(\frac{3}{4}\right)^2\left(1-\left(\frac{3}{4}\right)+\left(\frac{3}{4}\right)^2-...\right)=\left(\frac{3}{4}\right)^2\left(\frac{1}{4}+S\right)\Rightarrow\frac{7}{16}S=\frac{9}{64}\Rightarrow S=...$ | M1 | 3.1a |
| | $=\frac{9}{28}$ * | A1* | 1.1b |
| | | **(3 marks)** | |

**Notes**

B1: Deduces the correct value of the **first** term or the common ratio. The correct first term can be

seen as part of them writing down the sequence but must be the **first** term.
M1: Recognises that the series is infinite geometric and applies the sum to infinity GP formula with $a = \frac{9}{16}$ and $r = \pm\frac{3}{4}$
A1*: Correct proof

**Alternative 1:**
B1: Deduces the correct value for the sum to infinity (starting at $n = 1$) or the common ratio
M1: Calculates the required value by subtracting the first term from their sum to infinity
A1*: Correct proof

**Alternative 2:**
B1: Deduces the correct value of the **first** term or the common ratio.
M1: Splits the series into "odds" and "evens", attempts the sum of both parts and calculates the required value by adding both sums
A1*: Correct proof

**Alternative 3:**
B1: Deduces the correct value of the **first** term
M1: A complete method by taking out the first term, expresses the rhs in terms of the original sum and rearranges for "S"
A1*: Correct proof

| Question | Scheme | Marks | AOs |
|---|---|---|---|
| **3(a)** | Uses the common ratio $\dfrac{5+2\sin\theta}{12\cos\theta}=\dfrac{6\tan\theta}{5+2\sin\theta}$ o.e. | M1 | 3.1a |
| | Cross multiplies and uses $\tan\theta\times\cos\theta=\sin\theta$ <br> $(5+2\sin\theta)^2=6\times12\sin\theta$ | dM1 | 1.1b |
| | Proceeds to given answer $25+20\sin\theta+4\sin^2\theta=72\sin\theta$ <br> $\Rightarrow 4\sin^2\theta-52\sin\theta+25=0$ * | A1* | 2.1 |
| | | **(3)** | |
| **(a) Alt** | **(a) Alternative example:** | | |
| | Uses the common ratio $12r\cos\theta=5+2\sin\theta,\ 12r^2\cos\theta=6\tan\theta$ <br> $\Rightarrow 12\cos\theta\left(\dfrac{5+2\sin\theta}{12\cos\theta}\right)^2=6\tan\theta$ | M1 | 3.1a |
| | Multiplies up and uses $\tan\theta\times\cos\theta=\sin\theta$ <br> $(5+2\sin\theta)^2=6\tan\theta\times12\cos\theta=72\sin\theta$ | dM1 | 1.1b |
| | Proceeds to given answer $25+20\sin\theta+4\sin^2\theta=72\sin\theta$ <br> $\Rightarrow 4\sin^2\theta-52\sin\theta+25=0$ * | A1* | 2.1 |
| | | **(3)** | |
| **(b)** | $4\sin^2\theta-52\sin\theta+25=0\Rightarrow\sin\theta=\dfrac{1}{2}\left(,\dfrac{25}{2}\right)$ | M1 | 1.1b |
| | $\theta=\dfrac{5\pi}{6}$ | A1 | 1.2 |
| | | **(2)** | |
| **(c)** | Attempts a value for either $a$ or $r$ <br> e.g. $a=12\cos\theta=\ 12\times-\dfrac{\sqrt{3}}{2}$ **or** $r=\dfrac{5+2\sin\theta}{12\cos\theta}=\dfrac{5+2\times\frac{1}{2}}{12\times-\frac{\sqrt{3}}{2}}$ | M1 | 3.1a |
| | $"a"=-6\sqrt{3}$ **and** $"r"=-\dfrac{1}{\sqrt{3}}$ o.e. | A1 | 1.1b |
| | Uses $S_\infty=\dfrac{a}{1-r}=\dfrac{-6\sqrt{3}}{1+\frac{1}{\sqrt{3}}}$ | dM1 | 2.1 |
| | Rationalises denominator <br> $S_\infty=\dfrac{-6\sqrt{3}}{1+\frac{1}{\sqrt{3}}}=\dfrac{-18}{\sqrt{3}+1}\times\dfrac{\sqrt{3}-1}{\sqrt{3}-1}$ | ddM1 | 1.1b |
| | $(S_\infty=)9(1-\sqrt{3})$ | A1 | 2.1 |
| | | **(5)** | |
| | | | **(10 marks)** |
| **Notes:** | | | |

(a)

**M1**: For the key step in using the ratio of $\dfrac{a_2}{a_1}=\dfrac{a_3}{a_2}$

**dM1**: Cross multiplies and uses $\tan\theta\times\cos\theta=\sin\theta$

**A1***: Proceeds to the given answer including the " = 0" with no errors and sufficient working shown.

**Alternative:**

**M1**: Expresses the 2nd and 3rd terms in terms of the first term and the common ratio and eliminates "r"

**dM1**: Multiplies up and uses $\tan\theta \times \cos\theta = \sin\theta$

**A1***: Proceeds to the given answer including the " = 0" with no errors and sufficient working shown.

Other approaches may be seen in (a) and can be marked in a similar way e.g. M1 for correctly obtaining an equation in $\theta$ using the GP, M1 for applying $\tan\theta \times \cos\theta = \sin\theta$ or equivalent and eliminating fractions, A1 as above

| | |
|---|---|
| Example: $u_2 = \dfrac{u_1 \times u_3}{u_2} \Rightarrow 5 + 2\sin\theta = \dfrac{12\cos\theta \times 6\tan\theta}{5 + 2\sin\theta}$ | **M1** |
| $\Rightarrow (5 + 2\sin\theta)^2 = 72\sin\theta$ | **dM1** |
| $25 + 20\sin\theta + 4\sin^2\theta = 72\sin\theta$ | |
| $\Rightarrow 4\sin^2\theta - 52\sin\theta + 25 = 0 \quad *$ | **A1** |

(b)

**M1**: Attempts to solve $4\sin^2\theta - 52\sin\theta + 25 = 0$. Must be clear they have found $\sin\theta$ and not e.g. just $x$ from $4x^2 - 52x + 25 = 0$. Working does not need to be seen but see general guidance for solving a 3TQ if necessary. Note that the $\frac{25}{2}$ does not need to be seen.

**A1**: $\theta = \frac{5\pi}{6}$ and no other values unless they are rejected or the $\frac{5\pi}{6}$ clearly selected here and not in (c)

A minimum requirement in (b) is e.g. $\sin\theta = \frac{1}{2}, \quad \theta = \frac{5\pi}{6}$

Do **not** allow 150° for $\frac{5\pi}{6}$

**PTO for the notes to part (c)**

(c) Allow full marks in (c) if e.g. $\theta = \frac{\pi}{6}$ is their answer to (b) but $\theta = \frac{5\pi}{6}$ is used here.

or if e.g. $\theta = \frac{5\pi}{6}$ is their answer to (b) but $\theta = \frac{\pi}{6}$ is used here allow the M marks only.

**M1**: For attempting a value (exact or decimal) for either $a$ or $r$ using **their** $\theta$

E.g. $a = 12\cos\theta = \left(12\times -\frac{\sqrt{3}}{2}\right)$ or $r = \frac{5+2\sin\theta}{12\cos\theta} = \left(\frac{5+2\times\frac{1}{2}}{12\times -\frac{\sqrt{3}}{2}}\right)$ oe e.g. $r = \frac{6\tan\theta}{5+2\sin\theta} = \left(\frac{6\times -\frac{1}{\sqrt{3}}}{5+2\times\frac{1}{2}}\right)$

**A1**: Finds both $a = -6\sqrt{3}$ and $r = -\frac{1}{\sqrt{3}}$ which can be left unsimplified but $\sin\theta = \frac{1}{2}, \cos\theta = -\frac{\sqrt{3}}{2}$

and $\tan\theta = -\frac{\sqrt{3}}{3}$ (if required) must have been used.

**dM1**: Uses both **values** of "$a$" and "$r$" with the equation $S_\infty = \frac{a}{1-r} = \frac{-6\sqrt{3}}{1+\frac{1}{\sqrt{3}}}$ to create an expression

involving surds where $a$ and $r$ have come from appropriate work and $|r| < 1$

Depends on the first method mark.

**ddM1**: Rationalises denominator. The denominator must be of the form $p \pm q\sqrt{3}$ oe e.g. $p + \frac{q}{\sqrt{3}}$

Depends on both previous method marks.

Note that stating e.g. $\frac{k}{p+q\sqrt{3}} \times \frac{p-q\sqrt{3}}{p-q\sqrt{3}}$ or $\frac{k}{p+\frac{q}{\sqrt{3}}} \times \frac{p-\frac{q}{\sqrt{3}}}{p-\frac{q}{\sqrt{3}}}$ is sufficient.

**A1**: Obtains $(S_\infty =)\, 9(1-\sqrt{3})$

Note that full marks are available in (c) for the use of $\theta = 150°$

Note also that marks may be implied in (c) by e.g.

$$S_\infty = \frac{a}{1-r} = \frac{12\cos\theta}{1-\frac{5+2\sin\theta}{12\cos\theta}} = \frac{144\cos^2\theta}{12\cos\theta - 5 - 2\sin\theta} = \frac{144\cos^2\frac{5\pi}{6}}{12\cos\frac{5\pi}{6} - 5 - 2\sin\frac{5\pi}{6}}$$

$$= \frac{108}{-6-6\sqrt{3}} = \frac{108}{-6-6\sqrt{3}} \times \frac{-6+6\sqrt{3}}{-6+6\sqrt{3}} = \frac{-648+648\sqrt{3}}{-72} = 9(1-\sqrt{3})$$

Scores M1A1 implied dM1 ddM1 A1

**See next page for some other cases in (c) and how to mark them:**

$$S_\infty = \frac{a}{1-r} = \frac{12\cos\frac{5\pi}{6}}{1-\frac{5+2\sin\frac{5\pi}{6}}{12\cos\frac{5\pi}{6}}} \quad \text{or e.g.} \quad S_\infty = \frac{a}{1-r} = \frac{12\cos\frac{\pi}{6}}{1-\frac{5+2\sin\frac{\pi}{6}}{12\cos\frac{\pi}{6}}}$$

And nothing else

scores M1A0dM1ddM0A0

$$S_\infty = \frac{a}{1-r} = \frac{12\cos\frac{5\pi}{6}}{1-\frac{5+2\sin\frac{5\pi}{6}}{12\cos\frac{5\pi}{6}}} = 9\left(1-\sqrt{3}\right)$$

Scores M1A1dM1ddM0A0

$$S_\infty = \frac{a}{1-r} = \frac{12\cos\frac{\pi}{6}}{1-\frac{5+2\sin\frac{\pi}{6}}{12\cos\frac{\pi}{6}}} = 9\left(1+\sqrt{3}\right)$$

Scores M1A0dM1ddM0A0

$S_\infty = 9\left(1-\sqrt{3}\right)$ with no working scores no marks

| Question | Scheme | Marks | AOs |
|---|---|---|---|
| 4(a) | $\frac{12-3k}{3k+4}=\frac{k+16}{12-3k}$ | M1 | 3.1a |
| | $3k^2-62k+40=0$ * | A1* | 1.1b |
| | | (2) | |
| (b)(i) | $3k^2-62k+40=0 \Rightarrow k=...$ | M1 | 1.1b |
| | States $k=20$ and gives a reason e.g. that this gives a values of $r$ such that $\lvert r\rvert<1$ | A1 | 3.2a |
| (ii) | $a=64$ **and** $r=-\frac{3}{4}$ (or allow $a=6$ **and** $r=\frac{5}{3}$ ) | B1 | 1.1b |
| | $S_\infty=\frac{"64"}{1-"\left(-\frac{3}{4}\right)"}=...$ | M1 | 3.1a |
| | $S_\infty=\frac{256}{7}$ | A1 | 1.1b |
| | | (5) | |
| | | **(7 marks)** | |

**Notes**

**(a)**

M1: Forms a correct equation linking the three terms. Condone invisible brackets if implied by further work.
Possible equations below (which are not exhaustive) should use $n$th term or sum of series formulae

e.g. $\frac{12-3k}{3k+4}=\frac{k+16}{12-3k}$ or $\left(\frac{12-3k}{3k+4}\right)^2=\frac{k+16}{3k+4}$ or $(12-3k)^2=(3k+4)(k+16)$ or

$(3k+4)\left(\frac{k+16}{12-3k}\right)=12-3k$ or $(12-3k)\left(\frac{12-3k}{k+16}\right)=k+16$ or

$$3k+4+12-3k+k+16=\frac{(3k+4)\left(1-\left(\frac{k+16}{12-3k}\right)^3\right)}{1-\frac{k+16}{12-3k}}$$ (sum of three terms)

A1*: Achieves the given quadratic with no errors including invisible brackets. It cannot be for just proceeding in one step from the starting equation to the given answer and usually will involve attempting to multiply out brackets or dealing with any fractions.

**(b)(i)**

M1: Attempts to solve the given quadratic achieving at least one value for $k$. Usual rules apply for solving a quadratic and this may be achieved directly from a calculator. (May also be implied by $\frac{2}{3}$)

A1: 20 and gives correct reasoning (**if $r$ is found anywhere in part (i) then it must be correct**):
e.g. 20 since $|r|<1$. e.g. since $|r|=0.75<1$

e.g. by listing **at least two consecutive** terms for $k=20$ (must be correct) e.g. 64, -48 do not withhold this mark if they proceed to make a comment e.g. "the numbers are getting smaller" as we are condoning this to mean they are referring to the magnitude of the numbers

e.g. when $k=20$, $r=-\frac{3}{4}$ o.e. which is between 1 and -1 (condone "it is smaller than 1").

Do not accept a reason on its own which is just simply stating that the sequence is converging or equivalent such as "spiralling".

Allow reasoning which excludes $k=\frac{2}{3}$ e.g. $r=\frac{5}{3}$ which is greater than 1.

**(ii)**

**Work may be seen in part (i), but must be used in part (ii) to score.**

B1: $a=64$ **and** $r=-\frac{3}{4}$ o.e. (or allow $a=6$ **and** $r=\frac{5}{3}$ o.e.) May be implied by later work or a correct calculation using these values to find $S_\infty$

M1: A full attempt to find $S_\infty$ by using their value of $k$ to reach a value for $r$ such that $|r|<1$ and a value for $a$. Condone sign slips in their calculations of $a$ and $r$ only. You may need to check this by substituting in their value for $k$ if no calculations are seen.

They must substitute these values in to $\frac{a}{1-r}$ correctly so e.g. $a=64,\ r=-\frac{3}{4} \Rightarrow S_\infty = \frac{64}{1-\frac{3}{4}}$ is M0.

They cannot just substitute in their $k$ as $r$ in the formula.
Do not allow attempts to manually calculate the values of lots of terms for this mark as this would not lead to the answer. $\sum_{n=1}^{\infty} 64\times\left(-\frac{3}{4}\right)^{n-1}$ on its own is M0.

A1: $\frac{256}{7}$ cao. (Do not allow 36.6 as this is not $S_\infty$) isw after a correct exact answer is seen.